

# Rearranging equations

## A LEVEL LINKS

**Scheme of work:** 6a. Definition, differentiating polynomials, second derivatives

**Textbook:** Pure Year 1, 12.1 Gradients of curves

## Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

## Examples

**Example 1** Make  $t$  the subject of the formula  $v = u + at$ .

$v = u + at$ $v - u = at$ $t = \frac{v - u}{a}$	<ol style="list-style-type: none"> <li>Get the terms containing <math>t</math> on one side and everything else on the other side.</li> <li>Divide throughout by <math>a</math>.</li> </ol>
---	--

**Example 2** Make  $t$  the subject of the formula  $r = 2t - \pi t$ .

$r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \frac{r}{2 - \pi}$	<ol style="list-style-type: none"> <li>All the terms containing <math>t</math> are already on one side and everything else is on the other side.</li> <li>Factorise as <math>t</math> is a common factor.</li> <li>Divide throughout by <math>2 - \pi</math>.</li> </ol>
---	--

**Example 3** Make  $t$  the subject of the formula  $\frac{t+r}{5} = \frac{3t}{2}$ .

$\frac{t+r}{5} = \frac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \frac{2r}{13}$	<ol style="list-style-type: none"> <li>Remove the fractions first by multiplying throughout by 10.</li> <li>Get the terms containing <math>t</math> on one side and everything else on the other side and simplify.</li> <li>Divide throughout by 13.</li> </ol>
---	--

**Example 4** Make  $t$  the subject of the formula  $r = \frac{3t+5}{t-1}$ .

$r = \frac{3t+5}{t-1}$ $r(t-1) = 3t+5$ $rt - r = 3t+5$ $rt - 3t = 5+r$ $t(r-3) = 5+r$ $t = \frac{5+r}{r-3}$	<ol style="list-style-type: none"> <li><b>1</b> Remove the fraction first by multiplying throughout by <math>t-1</math>.</li> <li><b>2</b> Expand the brackets.</li> <li><b>3</b> Get the terms containing <math>t</math> on one side and everything else on the other side.</li> <li><b>4</b> Factorise the LHS as <math>t</math> is a common factor.</li> <li><b>5</b> Divide throughout by <math>r-3</math>.</li> </ol>
---	--

## Practice

Change the subject of each formula to the letter given in the brackets.

- |  |  |  |
|--|--|--|
| <b>1</b> $C = \pi d$ [ $d$ ]                       | <b>2</b> $P = 2l + 2w$ [ $w$ ]           | <b>3</b> $D = \frac{S}{T}$ [ $T$ ]       |
| <b>4</b> $p = \frac{q-r}{t}$ [ $t$ ]               | <b>5</b> $u = at - \frac{1}{2}t$ [ $t$ ] | <b>6</b> $V = ax + 4x$ [ $x$ ]           |
| <b>7</b> $\frac{y-7x}{2} = \frac{7-2y}{3}$ [ $y$ ] | <b>8</b> $x = \frac{2a-1}{3-a}$ [ $a$ ]  | <b>9</b> $x = \frac{b-c}{d}$ [ $d$ ]     |
| <b>10</b> $h = \frac{7g-9}{2+g}$ [ $g$ ]           | <b>11</b> $e(9+x) = 2e+1$ [ $e$ ]        | <b>12</b> $y = \frac{2x+3}{4-x}$ [ $x$ ] |

**13** Make  $r$  the subject of the following formulae.

- |                        |                                   |                           |                                     |
|------------------------|-----------------------------------|---------------------------|-------------------------------------|
| <b>a</b> $A = \pi r^2$ | <b>b</b> $V = \frac{4}{3}\pi r^3$ | <b>c</b> $P = \pi r + 2r$ | <b>d</b> $V = \frac{2}{3}\pi r^2 h$ |
|------------------------|-----------------------------------|---------------------------|-------------------------------------|

**14** Make  $x$  the subject of the following formulae.

- |   |  |
|---|--|
| <b>a</b> $\frac{xy}{z} = \frac{ab}{cd}$ | <b>b</b> $\frac{4\pi cx}{d} = \frac{3z}{py^2}$ |
|---|--|

**15** Make  $\sin B$  the subject of the formula  $\frac{a}{\sin A} = \frac{b}{\sin B}$

**16** Make  $\cos B$  the subject of the formula  $b^2 = a^2 + c^2 - 2ac \cos B$ .

## Extend

**17** Make  $x$  the subject of the following equations.

- |                                    |   |
|------------------------------------|---|
| <b>a</b> $\frac{p}{q}(sx+t) = x-1$ | <b>b</b> $\frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$ |
|------------------------------------|---|

## Answers

$$1 \quad d = \frac{C}{\pi}$$

$$2 \quad w = \frac{P-2l}{2}$$

$$3 \quad T = \frac{S}{D}$$

$$4 \quad t = \frac{q-r}{p}$$

$$5 \quad t = \frac{2u}{2a-1}$$

$$6 \quad x = \frac{V}{a+4}$$

$$7 \quad y = 2 + 3x$$

$$8 \quad a = \frac{3x+1}{x+2}$$

$$9 \quad d = \frac{b-c}{x}$$

$$10 \quad g = \frac{2h+9}{7-h}$$

$$11 \quad e = \frac{1}{x+7}$$

$$12 \quad x = \frac{4y-3}{2+y}$$

$$13 \quad \mathbf{a} \quad r = \sqrt{\frac{A}{\pi}}$$

$$\mathbf{b} \quad r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$\mathbf{c} \quad r = \frac{P}{\pi+2}$$

$$\mathbf{d} \quad r = \sqrt{\frac{3V}{2\pi h}}$$

$$14 \quad \mathbf{a} \quad x = \frac{abz}{cdy}$$

$$\mathbf{b} \quad x = \frac{3dz}{4\pi cpy^2}$$

$$15 \quad \sin B = \frac{b \sin A}{a}$$

$$16 \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$17 \quad \mathbf{a} \quad x = \frac{q+pt}{q-ps}$$

$$\mathbf{b} \quad x = \frac{3py + 2pqy}{3p - apq} = \frac{y(3+2q)}{3 - aq}$$