

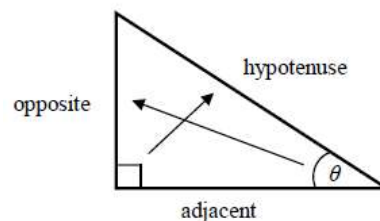
Trigonometry in right-angled triangles

A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

Key points

- In a right-angled triangle:
 - the side opposite the right angle is called the hypotenuse
 - the side opposite the angle θ is called the opposite
 - the side next to the angle θ is called the adjacent.

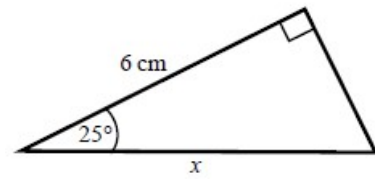


- In a right-angled triangle:
 - the ratio of the opposite side to the hypotenuse is the sine of angle θ , $\sin \theta = \frac{\text{opp}}{\text{hyp}}$
 - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ , $\cos \theta = \frac{\text{adj}}{\text{hyp}}$
 - the ratio of the opposite side to the adjacent side is the tangent of angle θ , $\tan \theta = \frac{\text{opp}}{\text{adj}}$
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: \sin^{-1} , \cos^{-1} , \tan^{-1} .
- The sine, cosine and tangent of some angles may be written exactly.

| | 0 | 30° | 45° | 60° | 90° |
|-----|---|----------------------|----------------------|----------------------|-----|
| sin | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| cos | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| tan | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | |

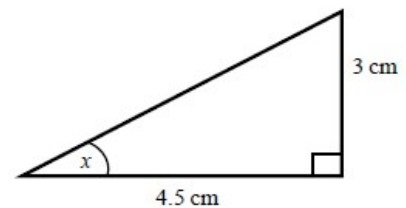
Examples

Example 1 Calculate the length of side x .
Give your answer correct to 3 significant figures.



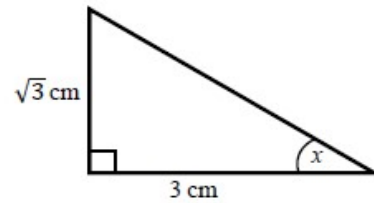
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| <p> $\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\cos 25^\circ = \frac{6}{x}$ $x = \frac{6}{\cos 25^\circ}$ $x = 6.620\ 267\ 5\dots$ $x = 6.62\ \text{cm}$ </p> | <ol style="list-style-type: none"> 1 Always start by labelling the sides. 2 You are given the adjacent and the hypotenuse so use the cosine ratio. 3 Substitute the sides and angle into the cosine ratio. 4 Rearrange to make x the subject. 5 Use your calculator to work out $6 \div \cos 25^\circ$. 6 Round your answer to 3 significant figures and write the units in your answer. |
|--|--|

Example 2 Calculate the size of angle x .
Give your answer correct to 3 significant figures.



| | |
|---|---|
| <p> $\tan \theta = \frac{\text{opp}}{\text{adj}}$ $\tan x = \frac{3}{4.5}$ $x = \tan^{-1}\left(\frac{3}{4.5}\right)$ $x = 33.690\ 067\ 5\dots$ $x = 33.7^\circ$ </p> | <ol style="list-style-type: none"> 1 Always start by labelling the sides. 2 You are given the opposite and the adjacent so use the tangent ratio. 3 Substitute the sides and angle into the tangent ratio. 4 Use \tan^{-1} to find the angle. 5 Use your calculator to work out $\tan^{-1}(3 \div 4.5)$. 6 Round your answer to 3 significant figures and write the units in your answer. |
|---|---|

Example 3 Calculate the exact size of angle x .

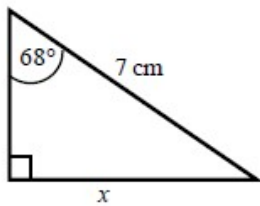


| | |
|--|---|
| <p> $\tan \theta = \frac{\text{opp}}{\text{adj}}$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30^\circ$ </p> | <ol style="list-style-type: none"> 1 Always start by labelling the sides. 2 You are given the opposite and the adjacent so use the tangent ratio. 3 Substitute the sides and angle into the tangent ratio. 4 Use the table from the key points to find the angle. |
|--|---|

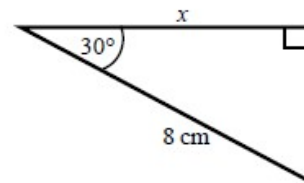
Practice

1 Calculate the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.

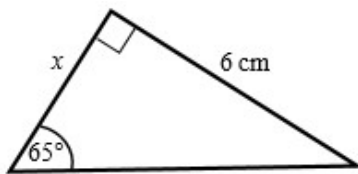
a



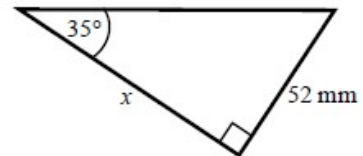
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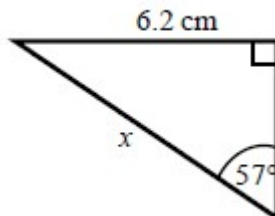
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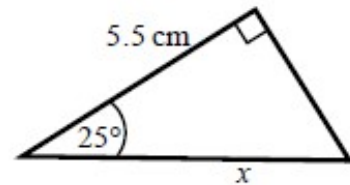
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e

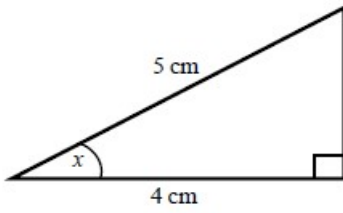


f

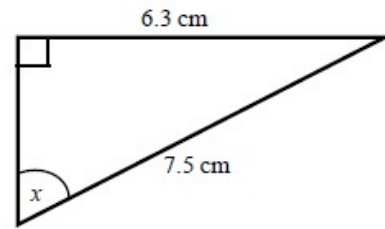


- 2 Calculate the size of angle x in each triangle. Give your answers correct to 1 decimal place.

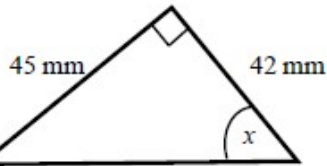
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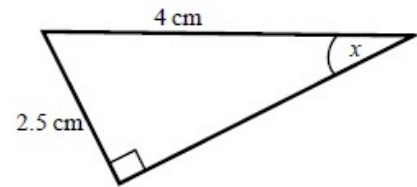
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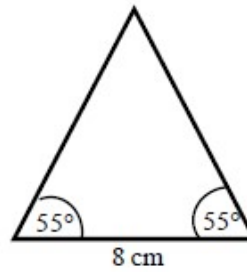
d



- 3 Work out the height of the isosceles triangle. Give your answer correct to 3 significant figures.

Hint:

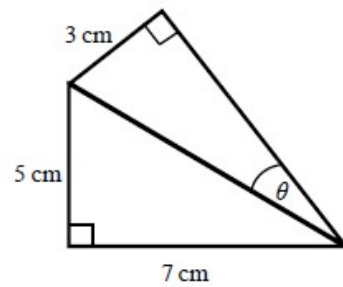
Split the triangle into two right-angled triangles.



- 4 Calculate the size of angle θ . Give your answer correct to 1 decimal place.

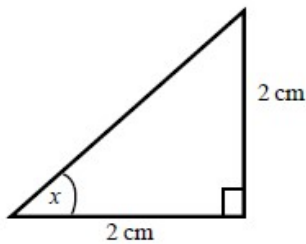
Hint:

First work out the length of the common side to both triangles, leaving your answer in surd form.

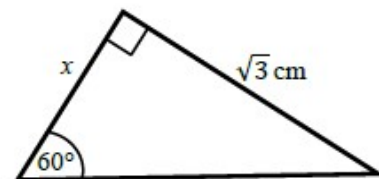


- 5 Find the exact value of x in each triangle.

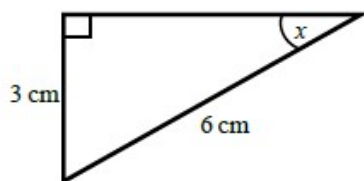
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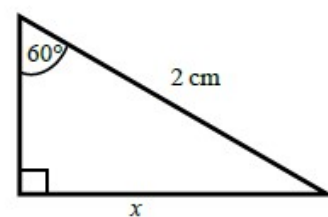
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d



The cosine rule

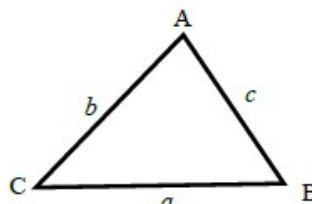
A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

Textbook: Pure Year 1, 9.1 The cosine rule

Key points

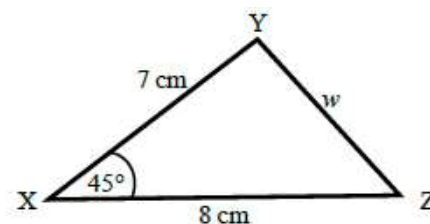
- a is the side opposite angle A .
 b is the side opposite angle B .
 c is the side opposite angle C .



- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc \cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

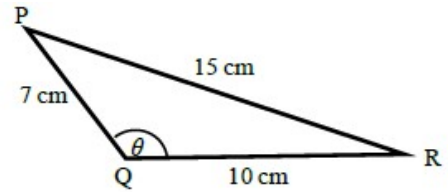
Examples

Example 4 Work out the length of side w .
Give your answer correct to 3 significant figures.



| | |
|---|---|
| <p> $a^2 = b^2 + c^2 - 2bc \cos A$ $w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45^\circ$ $w^2 = 33.804\ 040\ 51\dots$ $w = \sqrt{33.804\ 040\ 51}$ $w = 5.81 \text{ cm}$ </p> | <ol style="list-style-type: none"> 1 Always start by labelling the angles and sides. 2 Write the cosine rule to find the side. 3 Substitute the values a, b and A into the formula. 4 Use a calculator to find w^2 and then w. 5 Round your final answer to 3 significant figures and write the units in your answer. |
|---|---|

Example 5 Work out the size of angle θ .
Give your answer correct to 1 decimal place.

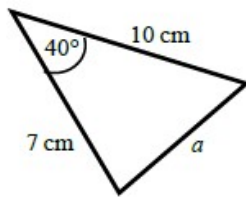


| | |
|---|--|
| $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos \theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$ $\cos \theta = \frac{-76}{140}$ $\theta = 122.878\ 349\dots$ $\theta = 122.9^\circ$ | <ol style="list-style-type: none"> 1 Always start by labelling the angles and sides. 2 Write the cosine rule to find the angle. 3 Substitute the values a, b and c into the formula. 4 Use \cos^{-1} to find the angle. 5 Use your calculator to work out $\cos^{-1}(-76 \div 140)$. 6 Round your answer to 1 decimal place and write the units in your answer. |
|---|--|

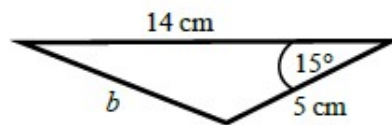
Practice

6 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

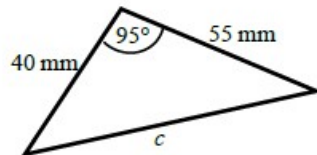
a



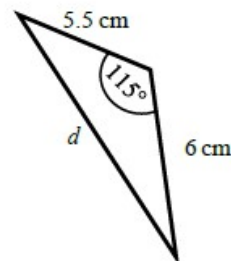
b



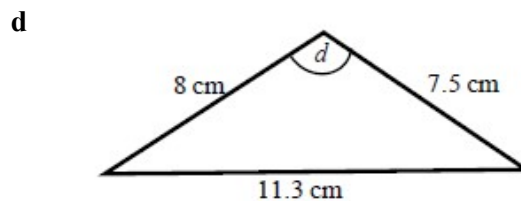
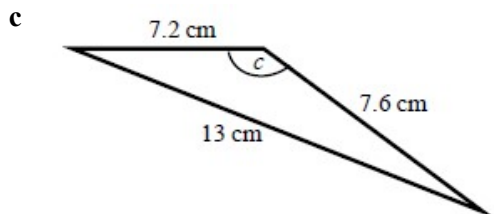
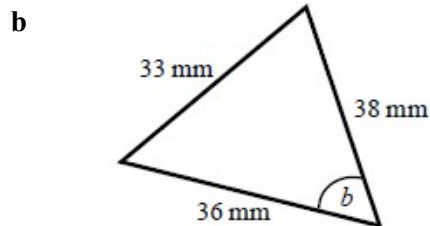
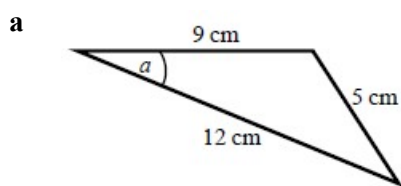
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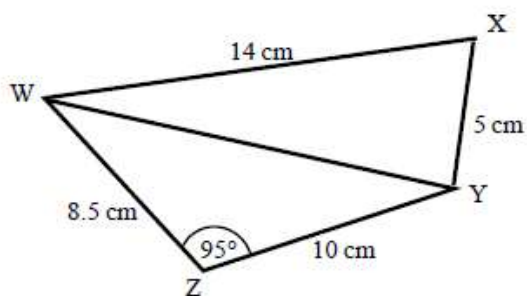
d



- 7 Calculate the angles labelled θ in each triangle. Give your answer correct to 1 decimal place.



- 8 a Work out the length of WY. Give your answer correct to 3 significant figures.
- b Work out the size of angle WXY. Give your answer correct to 1 decimal place.



The sine rule

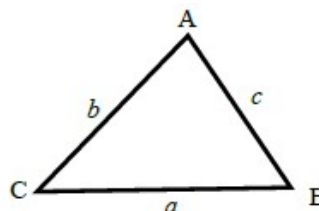
A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

Textbook: Pure Year 1, 9.2 The sine rule

Key points

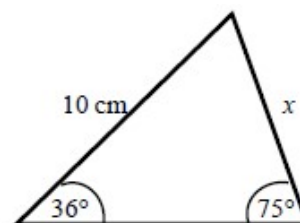
- a is the side opposite angle A .
- b is the side opposite angle B .
- c is the side opposite angle C .



- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

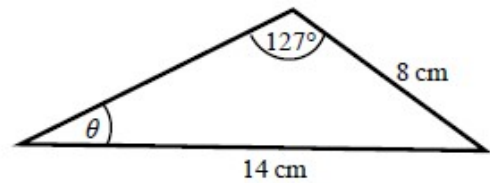
Examples

Example 6 Work out the length of side x .
Give your answer correct to 3 significant figures.



| | |
|--|--|
| $\frac{a}{\sin A} = \frac{b}{\sin B}$ $\frac{x}{\sin 36^\circ} = \frac{10}{\sin 75^\circ}$ $x = \frac{10 \times \sin 36^\circ}{\sin 75^\circ}$ $x = 6.09 \text{ cm}$ | <ol style="list-style-type: none"> 1 Always start by labelling the angles and sides. 2 Write the sine rule to find the side. 3 Substitute the values a, b, A and B into the formula. 4 Rearrange to make x the subject. 5 Round your answer to 3 significant figures and write the units in your answer. |
|--|--|

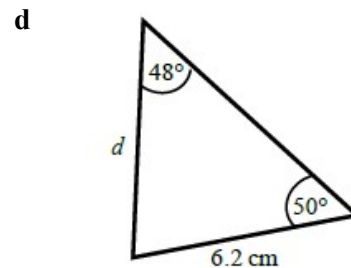
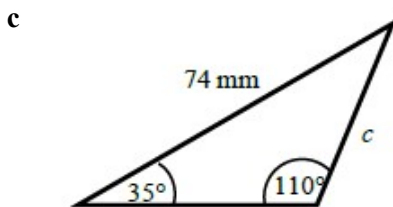
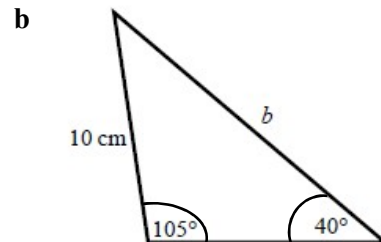
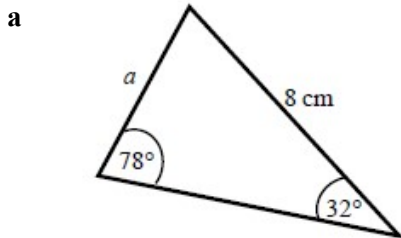
Example 7 Work out the size of angle θ .
Give your answer correct to 1 decimal place.



| | |
|--|---|
| $\frac{\sin A}{a} = \frac{\sin B}{b}$ $\frac{\sin \theta}{8} = \frac{\sin 127^\circ}{14}$ $\sin \theta = \frac{8 \times \sin 127^\circ}{14}$ $\theta = 27.2^\circ$ | <ol style="list-style-type: none"> 1 Always start by labelling the angles and sides. 2 Write the sine rule to find the angle. 3 Substitute the values a, b, A and B into the formula. 4 Rearrange to make $\sin \theta$ the subject. 5 Use \sin^{-1} to find the angle. Round your answer to 1 decimal place and write the units in your answer. |
|--|---|

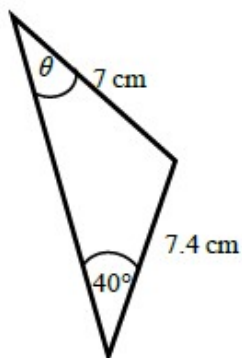
Practice

9 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

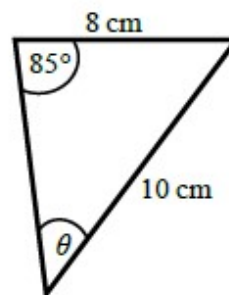


- 10 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

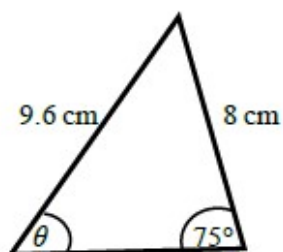
a



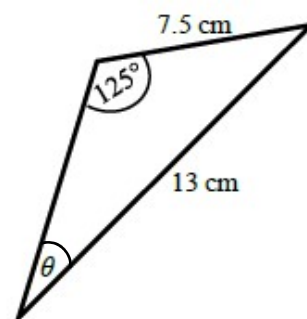
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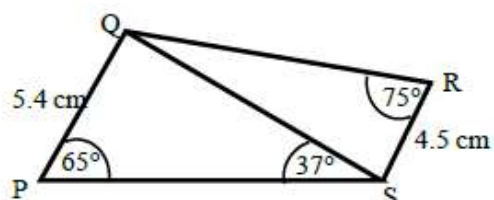
c



d



- 11 a Work out the length of QS.
Give your answer correct to 3 significant figures.
- b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.



Areas of triangles

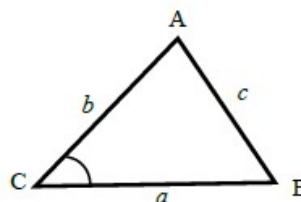
A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

Textbook: Pure Year 1, 9.3 Areas of triangles

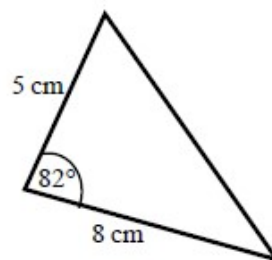
Key points

- a is the side opposite angle A .
 b is the side opposite angle B .
 c is the side opposite angle C .
- The area of the triangle is $\frac{1}{2}ab \sin C$.



Examples

Example 8 Find the area of the triangle.

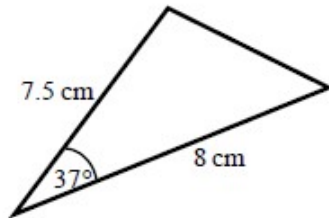


| | |
|---|---|
| <p>Area = $\frac{1}{2}ab \sin C$</p> <p>Area = $\frac{1}{2} \times 8 \times 5 \times \sin 82^\circ$</p> <p>Area = 19.805 361...</p> <p>Area = 19.8 cm²</p> | <ol style="list-style-type: none"> 1 Always start by labelling the sides and angles of the triangle. 2 State the formula for the area of a triangle. 3 Substitute the values of a, b and C into the formula for the area of a triangle. 4 Use a calculator to find the area. 5 Round your answer to 3 significant figures and write the units in your answer. |
|---|---|

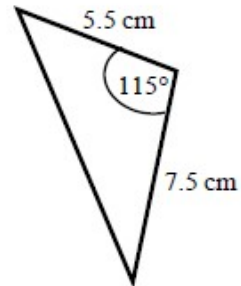
Practice

- 12 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

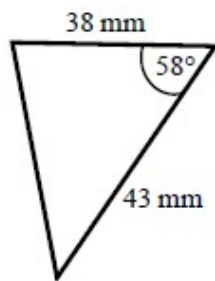
a



b



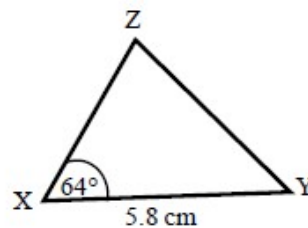
c



- 13 The area of triangle XYZ is 13.3 cm^2 .
Work out the length of XZ.

Hint:

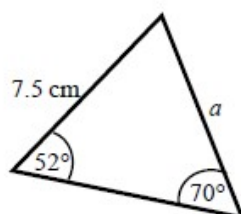
Rearrange the formula to make a side the subject.



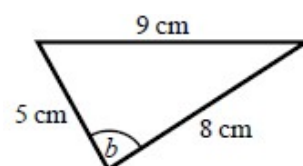
Extend

- 14 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

a



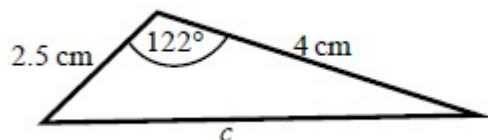
b



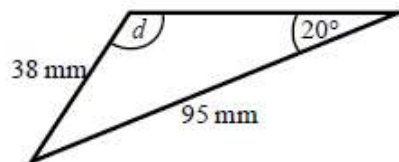
Hint:

For each one, decide whether to use the cosine or sine rule.

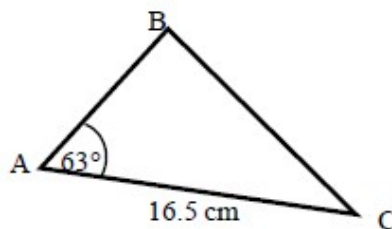
c



d



- 15 The area of triangle ABC is 86.7 cm^2 .
 Work out the length of BC.
 Give your answer correct to 3 significant figures.



Answers

- 1** **a** 6.49 cm **b** 6.93 cm **c** 2.80 cm
 d 74.3 mm **e** 7.39 cm **f** 6.07 cm
- 2** **a** 36.9° **b** 57.1° **c** 47.0° **d** 38.7°
- 3** 5.71 cm
- 4** 20.4°
- 5** **a** 45° **b** 1 cm **c** 30° **d** $\sqrt{3}$ cm
- 6** **a** 6.46 cm **b** 9.26 cm **c** 70.8 mm **d** 9.70 cm
- 7** **a** 22.2° **b** 52.9° **c** 122.9° **d** 93.6°
- 8** **a** 13.7 cm **b** 76.0°
- 9** **a** 4.33 cm **b** 15.0 cm **c** 45.2 mm **d** 6.39 cm
- 10** **a** 42.8° **b** 52.8° **c** 53.6° **d** 28.2°
- 11** **a** 8.13 cm **b** 32.3°
- 12** **a** 18.1 cm² **b** 18.7 cm² **c** 693 mm²
- 13** 5.10 cm
- 14** **a** 6.29 cm **b** 84.3° **c** 5.73 cm **d** 58.8°
- 15** 15.3 cm