

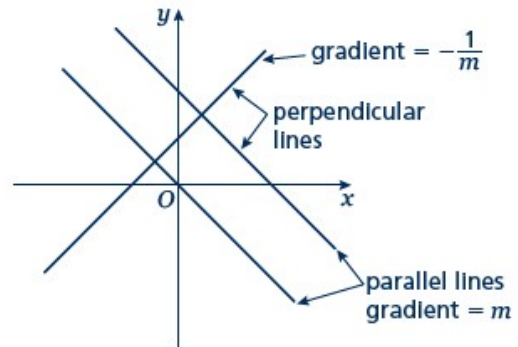
Parallel and perpendicular lines

A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\frac{1}{m}$.



Examples

Example 1 Find the equation of the line parallel to $y = 2x + 4$ which passes through the point $(4, 9)$.

$y = 2x + 4$ $m = 2$ $y = 2x + c$ $9 = 2 \times 4 + c$ $9 = 8 + c$ $c = 1$ $y = 2x + 1$	<ol style="list-style-type: none"> 1 As the lines are parallel they have the same gradient. 2 Substitute $m = 2$ into the equation of a straight line $y = mx + c$. 3 Substitute the coordinates into the equation $y = 2x + c$ 4 Simplify and solve the equation. 5 Substitute $c = 1$ into the equation $y = 2x + c$
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Example 2 Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point $(-2, 5)$.

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$ $y = -\frac{1}{2}x + c$ $5 = -\frac{1}{2} \times (-2) + c$ $5 = 1 + c$ $c = 4$ $y = -\frac{1}{2}x + 4$	<ol style="list-style-type: none"> 1 As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$. 2 Substitute $m = -\frac{1}{2}$ into $y = mx + c$. 3 Substitute the coordinates $(-2, 5)$ into the equation $y = -\frac{1}{2}x + c$ 4 Simplify and solve the equation. 5 Substitute $c = 4$ into $y = -\frac{1}{2}x + c$.
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Example 3 A line passes through the points (0, 5) and (9, -1).
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0, x_2 = 9, y_1 = 5 \text{ and } y_2 = -1$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{9 - 0}$ $= \frac{-6}{9} = -\frac{2}{3}$ $-\frac{1}{m} = \frac{3}{2}$ $y = \frac{3}{2}x + c$ $\text{Midpoint} = \left(\frac{0+9}{2}, \frac{5+(-1)}{2} \right) = \left(\frac{9}{2}, 2 \right)$ $2 = \frac{3}{2} \times \frac{9}{2} + c$ $c = -\frac{19}{4}$ $y = \frac{3}{2}x - \frac{19}{4}$	<ol style="list-style-type: none"> 1 Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line. 2 As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$. 3 Substitute the gradient into the equation $y = mx + c$. 4 Work out the coordinates of the midpoint of the line. 5 Substitute the coordinates of the midpoint into the equation. 6 Simplify and solve the equation. 7 Substitute $c = -\frac{19}{4}$ into the equation $y = \frac{3}{2}x + c$.
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Practice

- 1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1$ (3, 2)	b $y = 3 - 2x$ (1, 3)
c $2x + 4y + 3 = 0$ (6, -3)	d $2y - 3x + 2 = 0$ (8, 20)

- 2 Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point (-5, 3).

Hint

If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

- 3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

a $y = 2x - 6$ (4, 0)	b $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13)
c $x - 4y - 4 = 0$ (5, 15)	d $5y + 2x - 5 = 0$ (6, 7)

- 4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.
- a $(4, 3), (-2, -9)$ b $(0, 3), (-10, 8)$

Extend

- 5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$
 $y = 2x - 7$

b $y = 3x$
 $2x + y - 3 = 0$

c $y = 4x - 3$
 $4y + x = 2$

d $3x - y + 5 = 0$
 $x + 3y = 1$

e $2x + 5y - 1 = 0$
 $y = 2x + 7$

f $2x - y = 6$
 $6x - 3y + 3 = 0$

- 6 The straight line L_1 passes through the points A and B with coordinates $(-4, 4)$ and $(2, 1)$, respectively.

- a Find the equation of L_1 in the form $ax + by + c = 0$

The line L_2 is parallel to the line L_1 and passes through the point C with coordinates $(-8, 3)$.

- b Find the equation of L_2 in the form $ax + by + c = 0$

The line L_3 is perpendicular to the line L_1 and passes through the origin.

- c Find an equation of L_3

Answers

1 a $y = 3x - 7$

c $y = -\frac{1}{2}x$

b $y = -2x + 5$

d $y = \frac{3}{2}x + 8$

2 $y = -2x - 7$

3 a $y = -\frac{1}{2}x + 2$

c $y = -4x + 35$

b $y = 3x + 7$

d $y = \frac{5}{2}x - 8$

4 a $y = -\frac{1}{2}x$

b $y = 2x$

5 a Parallel**d** Perpendicular**b** Neither**e** Neither**c** Perpendicular**f** Parallel

6 a $x + 2y - 4 = 0$

b $x + 2y + 2 = 0$

c $y = 2x$