

# Surds and rationalising the denominator

## A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

## Key points

- A surd is the square root of a number that is not a square number, for example  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{5}$ , etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise  $\frac{a}{\sqrt{b}}$  you multiply the numerator and denominator by the surd  $\sqrt{b}$
- To rationalise  $\frac{a}{b + \sqrt{c}}$  you multiply the numerator and denominator by  $b - \sqrt{c}$

## Examples

**Example 1** Simplify  $\sqrt{50}$

$\begin{aligned}\sqrt{50} &= \sqrt{25 \times 2} \\ &= \sqrt{25} \times \sqrt{2} \\ &= 5 \times \sqrt{2} \\ &= 5\sqrt{2}\end{aligned}$	<ol style="list-style-type: none"> <li>1 Choose two numbers that are factors of 50. One of the factors must be a square number</li> <li>2 Use the rule <math>\sqrt{ab} = \sqrt{a} \times \sqrt{b}</math></li> <li>3 Use <math>\sqrt{25} = 5</math></li> </ol>
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**Example 2** Simplify  $\sqrt{147} - 2\sqrt{12}$

$\begin{aligned}\sqrt{147} - 2\sqrt{12} \\ &= \sqrt{49 \times 3} - 2\sqrt{4 \times 3} \\ \\ &= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3} \\ &= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3} \\ &= 7\sqrt{3} - 4\sqrt{3} \\ &= 3\sqrt{3}\end{aligned}$	<ol style="list-style-type: none"> <li>1 Simplify <math>\sqrt{147}</math> and <math>2\sqrt{12}</math>. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number</li> <li>2 Use the rule <math>\sqrt{ab} = \sqrt{a} \times \sqrt{b}</math></li> <li>3 Use <math>\sqrt{49} = 7</math> and <math>\sqrt{4} = 2</math></li> <li>4 Collect like terms</li> </ol>
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**Example 3** Simplify  $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$

$  \begin{aligned}  &(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2}) \\  &= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4} \\  &= 7 - 2 \\  &= 5  \end{aligned}  $	<ol style="list-style-type: none"> <li><b>1</b> Expand the brackets. A common mistake here is to write <math>(\sqrt{7})^2 = 49</math></li> <li><b>2</b> Collect like terms:  <math display="block">  \begin{aligned}  &amp;-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} \\  &amp;= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0  \end{aligned}  </math> </li> </ol>
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**Example 4** Rationalise  $\frac{1}{\sqrt{3}}$

$  \begin{aligned}  \frac{1}{\sqrt{3}} &= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\  &= \frac{1 \times \sqrt{3}}{\sqrt{9}} \\  &= \frac{\sqrt{3}}{3}  \end{aligned}  $	<ol style="list-style-type: none"> <li><b>1</b> Multiply the numerator and denominator by <math>\sqrt{3}</math></li> <li><b>2</b> Use <math>\sqrt{9} = 3</math></li> </ol>
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**Example 5** Rationalise and simplify  $\frac{\sqrt{2}}{\sqrt{12}}$

$  \begin{aligned}  \frac{\sqrt{2}}{\sqrt{12}} &= \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}} \\  &= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12} \\  &= \frac{2\sqrt{2}\sqrt{3}}{12} \\  &= \frac{\sqrt{2}\sqrt{3}}{6}  \end{aligned}  $	<ol style="list-style-type: none"> <li><b>1</b> Multiply the numerator and denominator by <math>\sqrt{12}</math></li> <li><b>2</b> Simplify <math>\sqrt{12}</math> in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number</li> <li><b>3</b> Use the rule <math>\sqrt{ab} = \sqrt{a} \times \sqrt{b}</math></li> <li><b>4</b> Use <math>\sqrt{4} = 2</math></li> <li><b>5</b> Simplify the fraction:  <math>\frac{2}{12}</math> simplifies to <math>\frac{1}{6}</math> </li> </ol>
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**Example 6** Rationalise and simplify  $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$ $= \frac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$ $= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$ $= \frac{6-3\sqrt{5}}{-1}$ $= 3\sqrt{5}-6$	<p><b>1</b> Multiply the numerator and denominator by <math>2-\sqrt{5}</math></p> <p><b>2</b> Expand the brackets</p> <p><b>3</b> Simplify the fraction</p> <p><b>4</b> Divide the numerator by <math>-1</math> Remember to change the sign of all terms when dividing by <math>-1</math></p>
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## Practice

**1** Simplify.

- a  $\sqrt{45}$   
 c  $\sqrt{48}$   
 e  $\sqrt{300}$   
 g  $\sqrt{72}$

- b  $\sqrt{125}$   
 d  $\sqrt{175}$   
 f  $\sqrt{28}$   
 h  $\sqrt{162}$

### Hint

One of the two numbers you choose at the start must be a square number.

**2** Simplify.

- a  $\sqrt{72} + \sqrt{162}$   
 c  $\sqrt{50} - \sqrt{8}$   
 e  $2\sqrt{28} + \sqrt{28}$

- b  $\sqrt{45} - 2\sqrt{5}$   
 d  $\sqrt{75} - \sqrt{48}$   
 f  $2\sqrt{12} - \sqrt{12} + \sqrt{27}$

### Watch out!

Check you have chosen the highest square number at the start.

**3** Expand and simplify.

- a  $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$   
 c  $(4 - \sqrt{5})(\sqrt{45} + 2)$

- b  $(3 + \sqrt{3})(5 - \sqrt{12})$   
 d  $(5 + \sqrt{2})(6 - \sqrt{8})$

4 Rationalise and simplify, if possible.

a  $\frac{1}{\sqrt{5}}$

b  $\frac{1}{\sqrt{11}}$

c  $\frac{2}{\sqrt{7}}$

d  $\frac{2}{\sqrt{8}}$

e  $\frac{2}{\sqrt{2}}$

f  $\frac{5}{\sqrt{5}}$

g  $\frac{\sqrt{8}}{\sqrt{24}}$

h  $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a  $\frac{1}{3-\sqrt{5}}$

b  $\frac{2}{4+\sqrt{3}}$

c  $\frac{6}{5-\sqrt{2}}$

## Extend

6 Expand and simplify  $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$

7 Rationalise and simplify, if possible.

a  $\frac{1}{\sqrt{9}-\sqrt{8}}$

b  $\frac{1}{\sqrt{x}-\sqrt{y}}$

## Answers

1 a  $3\sqrt{5}$   
 c  $4\sqrt{3}$   
 e  $10\sqrt{3}$   
 g  $6\sqrt{2}$

b  $5\sqrt{5}$   
 d  $5\sqrt{7}$   
 f  $2\sqrt{7}$   
 h  $9\sqrt{2}$

2 a  $15\sqrt{2}$   
 c  $3\sqrt{2}$   
 e  $6\sqrt{7}$

b  $\sqrt{5}$   
 d  $\sqrt{3}$   
 f  $5\sqrt{3}$

3 a  $-1$   
 c  $10\sqrt{5}-7$

b  $9-\sqrt{3}$   
 d  $26-4\sqrt{2}$

4 a  $\frac{\sqrt{5}}{5}$   
 c  $\frac{2\sqrt{7}}{7}$   
 e  $\sqrt{2}$   
 g  $\frac{\sqrt{3}}{3}$

b  $\frac{\sqrt{11}}{11}$   
 d  $\frac{\sqrt{2}}{2}$   
 f  $\sqrt{5}$   
 h  $\frac{1}{3}$

5 a  $\frac{3+\sqrt{5}}{4}$

b  $\frac{2(4-\sqrt{3})}{13}$

c  $\frac{6(5+\sqrt{2})}{23}$

6  $x-y$

7 a  $3+2\sqrt{2}$

b  $\frac{\sqrt{x}+\sqrt{y}}{x-y}$